39th CHAMPIONSHIP, 2025: MATHEMATICAL AND LOGIC GAMES
FBJM Quarter final

Information at http://www.fbjm.be

START FOR ALL PARTICIPANTS

1. TWO DATES (coefficient 1)

Matthew has these twelve cards. In 2025, the first date of the year that he can form using two cards for the day, two cards for the month and four for the year is January 13, 13 01 2025.

What will be the last date of 2025 that he can form using eight of the twelve cards?

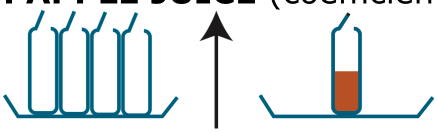
2. INTERSECTIONS (coefficient 2)

If we draw two circles and a line, we obtain a maximum of 6 points of intersection.

What is the maximum number of points of intersection that we would obtain if we drew two circles and two lines?

Note: We must count the intersections between two lines, between two circles, and between a line and a circle.

3. APPLE JUICE (coefficient 3)



A half-full bottle of apple juice weighs exactly the same as four identical empty bottles.

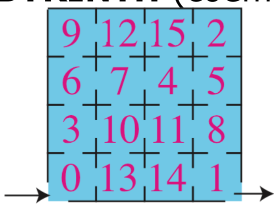
If we place a full bottle of this apple juice on the right pan of a scale, how many empty bottles would we need to place on the left pan to balance the scale?

4. AQUARIUM (coefficient 4)

In an aquarium live some octopus, which each have eight arms, and some starfish with five arms.

How many starfish are there in the aquarium, knowing that the total number of arms of all the animals is equal to 41?

5. LABYRINTH (coefficient 5)



In this maze, the rooms are numbered from 0 to 15. When you go from one room to another through a door, an alarm goes off except:

- if the number of the room you are entering is equal to that of the room you are leaving plus 3;
- or if the number of the room you are entering is equal to that of the room you are leaving minus 13.

You enter the maze through room no. 0 and exit through room no. 1.

How many rooms will you have gone through (including room 0 and room 1) if you have not triggered an alarm?

END FOR CE PARTICIPANTS

6. FOUR FRIENDS (coefficient 6)

Annabelle, Bertram, Clarisse and Damien are four friends.

Each of them is destined for a different specific career: archaeologist, bookkeeper, cardiologist, and Bertram wants to become a dentist.

Only one of these people is destined for the career that has the same initial as his/her first name, but it is not Annabelle. Besides, Annabelle would especially not want to work in the medical field.

What careers have Annabelle and Damien chosen?

7. BOXING MATCH (coefficient 7)
Place the numbers 1, 2, 3, 4, 5, 7 in the following boxes so that in any three boxes placed side by side, one of the numbers is equal to the sum of the other two, and the first digit is less than the last.

8. PLAY DATE (coefficient 8)

$$\begin{array}{r} \text{PLAY} \\ +\ \ \text{LA} \\ \hline =\ 2025 \end{array}$$

In this cryptarithm, different letters always replace different digits and the same digit is always replaced by the same letter. The first digit of a multi-digit number cannot be a 0.
What is the value of LA?

END FOR CM PARTICIPANTS

Problems 9 to 18: beware! For a problem to be completely solved, you must give both the number of solutions, AND give the solution if there is only one, or give any two correct solutions if there are more than one. For all problems that may have more than one solution, there is space for two answers on the answer sheet (but there may still be just one solution).

9. PLANET MATHS (coefficient 9)
On Planet Maths, a day does not last 24 hours like it does on Planet Earth. On a Mathsian's clock face, all the hours are arranged in a circle at equal intervals. The hour hand travels the same distance between 1 o'clock and 9 o'clock as it does between 10 o'clock and 2 o'clock.
How many hours are there in a day on this planet?

10. SUM-SUM-PRODUCT INTEGERS (coefficient 10)
A sum-sum-product integer is equal to the sum of the sum of its digits and the product of its digits. The number 59 is an example because (5+9) + (5 × 9) = 14 + 45 = 59.
How many two-digit sum-sum-product integers are there (counting 59)?

11. THREE SQUARES (coefficient 11)
Matthew drew three squares with sides measuring whole numbers of centimetres, two of which are identical. The sum of the areas of the three squares is equal to 2025 cm^2.
What is the perimeter (in cm) of the smallest square, or of one of them if they are identical?

END FOR C1 PARTICIPANTS

12. AVERAGES (coefficient 12)
25, A, B, 250, C, ...
In this sequence of numbers, each number starting from the second is the average of the two numbers that surround it.
What is the value of the number C?

13. COMPETITION SCORES (coef. 13)
In this competition, the name of which we will not mention, participants must answer 18 questions numbered from 1 to 18, their answer to each question being either correct or incorrect. Each person obtains a first score corresponding to the number of correct answers, and a second score corresponding to the sum of the numbers of the questions they answered correctly. In the event of a tie on the first score, the participants are separated by the second score. It so happens that in the last competition there was no tie after taking into account the two scores.
How many competitors were there in this competition, at most?

14. REACHING 2025 (coefficient 14)

We can build a sequence of whole numbers by adding to each number twice the sum of the digits that compose it. For example, starting from 1000, we obtain:

• 1st step: 1002 = 1000 + 2(1+ 0 + 0 + 0),

• 2nd step: 1008 = 1002 + 2(1+ 0 + 0 + 2),

• 3rd step: 1026 (= 1008 + 2(1+ 0 + 0 + 8), etc…

How many starting numbers strictly less than 2025 allow us to arrive at the number 2025?

END FOR C2 PARTICIPANTS

15. URNS AND BALLS (coefficient 15)

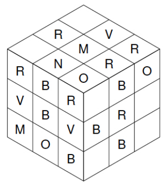
Each of the two players has a white urn containing two black balls and a black urn containing two white balls. In each round, the first player takes a ball at random from each of his urns and exchanges them, while the second player takes a ball at random from his white urn, puts it in his black urn, then takes a ball at random from his black urn and puts it in his white urn. The first player to end up with the white balls in his white urn and the black balls in his black urn wins. In the event of a tie, both players have won.

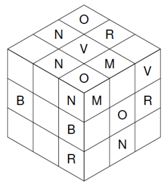
What is the probability that the first player wins?

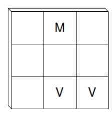
Give the answer as an irreducible fraction.

16. CUBACIOUS (coefficient 16)

In the last century, an archaeologist found a 3,000-year-old cube. She was able to determine that the cube had been made in the following way. With 27 small wooden cubes, a large cube was made, then one face was painted Red, one Blue, one Violet, one Navy, one Maroon and one Orange. Then, the cubes were mixed again and another large cube was made with them, so that only the painted faces of the small cubes are visible. Unfortunately, this cube disappeared in a fire shortly after its discovery. Only three photos remain, which unfortunately have lost some of their colours over time. Today the archaeologist's granddaughter is trying to reconstruct the original colours.







Help her find them by completing the third photo.

END FOR L1, GP PARTICIPANTS

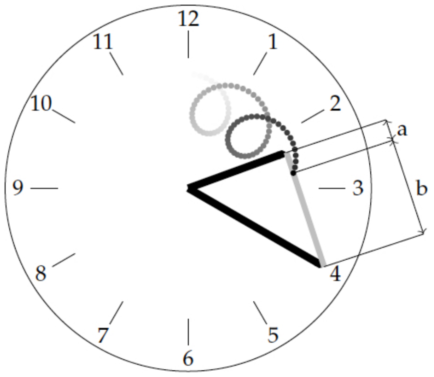
17. RUSSIAN DOLLS (coefficient 17)

Matilda sees a number of Russian dolls. She knows that there are 13 Russian dolls in total, numbered from 1, the smallest, to 13, the largest. She also knows that all the dolls she does not see are nested inside the dolls she sees. When one opens a doll, there is at most one other doll visible inside, which itself can contain another doll, etc. She wonders how the dolls are nested and realizes that there are 2025 possibilities.

What are the numbers of the dolls she sees?

Give the numbers in descending order.

18. TICK-TACK-TOCK (coefficient 18)



Tick and Tack are a bit crazy. Tick attaches the two ends of a rubber band to the ends of the hands of a clock, the hour hand measuring 2 cm and the minute hand 3 cm. Tack draws a black dot somewhere on the rubber band (but not at the ends). When the time advances, this black dot will move, the ratio a/b remaining constant (see the drawing). We see that the figure traced by the black dot intersects itself. Tick and Tack repeat the experiment and realize that this time the drawn figure no longer intersects itself.

What is the maximum a/b ratio when the figure does not intersect itself?

Write the answer as an irreducible fraction.

END FOR L2, HC PARTICIPANTS